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Atmospheric Absorption Measurements with a Microwave Radiometer[1]

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The absorption of microwave radiation in traversing the earth's atmosphere has been measured at three wave-lengths (1.00 cm, 1.25 cm, and 1.50 cm) in the region of a water-vapor absorption line. The measurement employs a sensitive radiometer to detect thermal radiation from the absorbing atmosphere. The theory of such measurements and the connection between absorption and thermal radiation are presented. The measured absorption together with water-vapor soundings of the atmosphere permits the calculation of the absorption coefficients at standard conditions (293°K, 1015 millibar). These are 0.011, 0.026, and 0.014 db/km/g H_2O/m^3 for the wave-lengths 1.00 cm, 1.25 cm, and 1.50 cm, respectively. These values are (50 percent) greater than those given by the theory of Van Vleck. The collision width of the line and its location are in better agreement with the theory and infra-red absorption measurement. It is also found that there is very little (<20°K) radiation from cosmic matter at the radiometer wave-lengths.

I. INTRODUCTION

THE absorption of centimeter, electromagnetic waves in atmospheric gases has received considerable attention recently. There are two known contributions to this absorption: oxygen, which has a band of resonance absorption lines in the region of $\frac{1}{2}$ cm superimposed on a weak continuum extending up to long wave-length and water vapor, which has a weak absorption line at approximately 1.3 cm, and a number of stronger lines below 0.2 cm, the far tails of which contribute to the absorption in the centimeter wave-length region. Both of these were predicted theoretically by Van Vleck[5] in 1942 and their absorption coefficients were calculated subject to experimental determinations of the line widths owing to collision broadening and some uncertainty in the location of the water vapor resonance. More recently, these absorptions have been measured[6] by several methods. The present paper describes atmospheric absorption measurements initiated by one of the authors in 1944 using a microwave radiometer[7] and is principally concerned with the water vapor absorption.

Except for the O_2 band, the atmospheric absorption is too small to be easily measured with path lengths which are available in the laboratory. This has led several observers[6] to employ resonant cavities. The present experiments make use of the entire atmosphere as an absorption path, measuring the thermal radiation which is emitted by the absorbing atmosphere in accordance with the Kirchhoff-Einstein law. The absorption is then calculated from the measured thermal radiation assuming local thermodynamic equilibrium in the atmosphere.

The microwave radiometers consisted of a directional antenna which was pointed at the sky and which was connected to a special microwave receiver which gave an indication of the thermal radiation intercepted by the antenna.[8] It is convenient to consider this radiation as originating in the effective terminating impedance of the antenna and to assign to this impedance a

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[5] J. H. Van Vleck, Radiation Laboratory Report 43-2 and Radiation Laboratory Report 664.

[6] J. A. Saxton, British Report RRBS 17, measured the absorption of superheated steam in a resonant cavity. J. W. Miller and R. S. Bender, Radiation Laboratory Report 729 measured the absorption in the atmosphere at 1.25 cm with radar techniques. Kellogg, Phys. Rev. 69, 694A (1946), has measured the absorption in moist air in the range 0.9 cm to 1.7 cm with a cavity resonator. R. Beringer, Radiation Laboratory Report 684, has measured the O_2 absorption in the range 0.5 to 0.6 cm with a wave guide absorption path.

[7] R. H. Dicke, Radiation Laboratory Report 787 and forthcoming article in *The Review of Scientific Instruments.*

[8] Such measurements have been made in the infra-red using somewhat different techniques. See, e.g., J. Strong, J. Opt. Soc. Am. 29, 520 (1938), J. Frank. Inst. 232, 2 (1941).

thermodynamic temperature t (°K) such that the corresponding "Johnson noise" is equal to the observed radiation.[9] If the radiation intercepted by the antenna is P ergs/sec. in the frequency range $\Delta\nu$ cycles/sec., this temperature is $t=P(k\Delta\nu)^{-1}$ °K. This will be called the noise temperature of the antenna or simply the *antenna temperature*. The connection between the antenna temperature and the thermodynamic temperatures of various absorbing systems will be discussed in the following paragraphs.

If the atmosphere is transparent at the signal frequency of the microwave receiver, the effective antenna termination will be the stars and other cosmic matter and the antenna temperature will be characteristic of this matter. At ordinary radio frequencies, the radiation from this matter, the so-called *cosmic noise*, is quite large.[10] However, at the frequencies in question the cosmic noise was found to be very small.

If there is absorption in the atmosphere, this contributes to the antenna temperature and, if the thermodynamic temperature and distribution of absorption in the atmosphere are known, the total fractional absorption in the atmosphere can be computed from a measurement of this contribution to the antenna temperature. In general, this total absorption is made up of contributions from various atmospheric layers, each at a different temperature and pressure. It is possible, however, to reduce these data to some standard condition (say 20°C and 1015 millibar) and to find the corresponding absorption coefficient if the distribution of the absorption in the atmosphere is known. If the variation of this absorption coefficient with wave-length is measured in a region of resonance, the location, strength, and collision-width of the resonance may be deduced.

II. THEORY OF THE MEASUREMENT

Consider a matched[11] antenna connected to a lossless transmission line which is terminated in a matched load (a load impedance equal to the characteristic impedance of the transmission line, Z_0). Let this be imbedded in an absorbing medium in thermodynamic equilibrium at temperature T and of infinite extent, and let Z_0 also be at temperature T. In the frequency range ν to $\nu+\Delta\nu$ the resistor radiates an average power $kT\Delta\nu$ ergs/sec. which in turn is radiated by the antenna. In order that the second law of thermodynamics be obeyed, it is necessary that the antenna intercept an equal amount of power. The absorbing medium, being a blackbody, radiates an amount of power

$$\frac{2h\nu^3}{c^2(e^{h\nu/kt}-1)}\Delta\nu d\Omega \quad (1)$$

into the solid angle $d\nu$ in the frequency range ν to $\nu+\Delta\nu$, as given by Planck's formula. This reduces to the Rayleigh-Jeans expression

$$\frac{2kT\nu^2}{c^2}\Delta\nu d\Omega \quad (2)$$

for $h\nu/k \ll T$ which is the case of interest. (At 2.4×10^{10} cycles/sec., $h\nu/k=1.14$°K as compared with atmospheric temperatures $T\sim300$°K.) The total radiation intercepted by the antenna and in turn absorbed by Z_0 is

$$P=\tfrac{1}{2}\int\frac{2kT\nu^2}{c^2}\Delta\nu\sigma(\theta,\phi)d\Omega \quad (3)$$

in the range $\Delta\nu$, where the factor $\frac{1}{2}$ is introduced because an antenna accepts a single polarization. $\sigma(\phi,\theta)$ is the absorption cross section of the antenna in spherical coordinates. It can be shown from the reciprocity theorem for antennas that

$$\int\sigma(\theta,\phi)d\Omega=c^2/\nu^2. \quad (4)$$

Thus

$$P=kT\Delta\nu, \quad (5)$$

and the thermal radiation emitted by Z_0 is just equal to that absorbed. The temperature T is the antenna temperature in this example. Hence the antenna temperature of any blackbody is just the thermodynamic temperature of that body. Equation (5) will be recognized as a generalization of the Johnson noise formula to a system including antennas.

[9] The available "Johnson noise" power from any impedance is $kT\Delta\nu$ ergs/sec. in the frequency range ν to $\nu+\Delta\nu$ cycles/sec., where k is Boltzmann's constant.

[10] K. G. Jansky, Proc. I.R.E. **20**, 1920 (1932); Proc. I.R.E. **21**, 1387 (1933); Proc. I.R.E. **23**, 1158 (1935); G. Reber; Proc. I.R.E. **28**, 68 (1940); Proc. I.R.E. **30**, 367 (1942); Astrophys. J. **91**, 621 (1940). K. Franz, Hoch. tech. u. Elek: akus. **59**, 1943 (1942).

[11] We use *matched* in the usual transmission line sense, e.g., a wave originating in the load and running toward the antenna is radiated into space without reflection.

When an antenna is pointed at an incompletely absorbing medium whose thermodynamic temperature is T, the antenna temperature is, of course, less than T. This is easily shown by the example of Fig. 1. Consider a matched antenna-transmission line-matched load system to be imbedded in a homogeneous, isotropic absorbing medium **1** at temperature T which is bounded at $x=l$ by a similar semi-infinite medium **2** at temperature T'. The antenna pattern, $\sigma(\theta, \phi)$, is assumed to be unidirectional along the x axis. The noise power originating in medium **2** and intercepted by the antenna is

$$kT'\Delta\nu e^{-\alpha 1}, \tag{6}$$

where α is the absorption coefficient of medium **1** (defined by $P(x)=e^{-\alpha x}P(x=0)$ for a wave running in the $+x$ direction). If $T'=T$, it is clear from (5) that the total intercepted power is $kT\Delta\nu$. Therefore the noise power originating in medium **1** and intercepted by the antenna is

$$kT\Delta\nu(1-e^{-\alpha 1}). \tag{7}$$

In general $T'\neq 0$ and the total noise power intercepted by the antenna is

$$P=k\Delta\nu[T-(T-T')e^{-\alpha 1}]. \tag{8}$$

The quantity $[T-(T-T')e^{-\alpha 1}]$ is the *antenna temperature*, t, of the system. The quantity $(1-e^{-\alpha 1})$ is the *fractional absorption*, a, for the path length l in medium **1** since

$$1-e^{-\alpha 1}=\frac{P(x=0)-P(x=1)}{P(x=0)} \tag{9}$$

for a wave running in the $+x$ direction.

The most important case of interest is for $T'=0$. Then the total noise power intercepted by the antenna is

$$P=kT\Delta\nu(1-e^{-\alpha 1}) \tag{10}$$

$$=kT\Delta\nu a, \tag{11}$$

and the antenna temperature is

$$t=Ta. \tag{12}$$

Thus if the temperature T of such a medium is known and if the antenna temperature t is measured one immediately has the total fractional absorption a for the path length l by using (12).

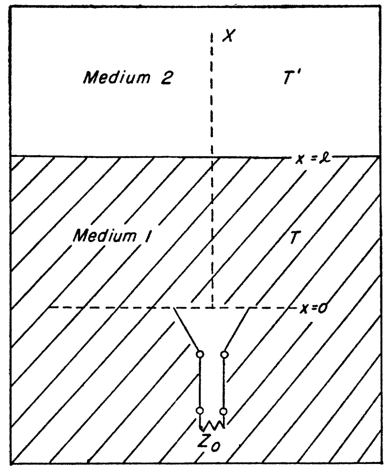

FIG. 1. The measurement of antenna temperatures in a composite absorbing system. Medium **1** is at temperature T(°K) and is bounded at $x=l$ by medium **2** which is at temperature T' and which extends to $x=\infty$.

One can also compute α if l is known. If the absorption is small $a\cong\alpha 1$.

In general the absorption coefficient, $\alpha(x)$, and the temperature, $T(x)$, of medium **1** may be functions of x. In the case of $T'\neq 0$ the total noise power intercepted by the antenna is

$$P=k\Delta\nu T' \exp\left[-\int_0^l \alpha(x)dx\right]$$

$$+k\Delta\nu\int_0^l \alpha(x)T(x)dx \exp\left[-\int_0^x \alpha(x)dx\right]. \tag{13}$$

In the measurements reported here the antenna temperature t is measured for the zenith direction and at several angles (48.2°, 60°, 66.5°) from the vertical. This is done for several reasons. First; by reason of its construction the apparatus measures the difference between t and the ambient temperature of the apparatus (say, 300°K). As the absorption is weak, t at the zenith is quite small (say, 30°K), and a small percentage error in the measurement gives a large error in t. The change in t with zenith angle can be measured with considerably greater precision and, under suitable conditions, can be used to compute the absorption. Also, this change in t with angle enables one to evaluate the contribution due to the cosmic noise.

An exact analysis of the antenna temperature as a function of tipping angle is easily made if we assume the atmosphere to be of uniform temperature throughout. Nothing need be assumed about the distribution of the absorption so long as it is horizontally stratified. We shall, for the moment, assume the cosmic noise to be zero. If the absorbing layer is of height h, the antenna temperature at the zenith ($\theta=0$) is, from (13),

$$t_0=T\int_0^h \alpha(x)e^{-\int_0^x \alpha(x)dx}dx, \quad (14)$$

and at a tipping angle θ, it is

$$t_\theta=T\sec\theta\int_0^h \alpha(x)e^{-\sec\theta\int_0^x \alpha(x)dx}dx. \quad (15)$$

From (14) and (15)

$$\frac{t_\theta-t_0}{T}=\left(1-\frac{t_0}{T}\right)-\left(1-\frac{t_0}{T}\right)^{\sec\theta}. \quad (16)$$

At $\theta=60°$, (16) can be solved for

$$\frac{t_0}{T}=\frac{1}{2}-\frac{1}{2}\left[1-4\left(\frac{t_{60}-t_0}{T}\right)\right]^{\frac{1}{2}}. \quad (17)$$

This quantity t_0/T is the fractional absorption a_0 along a path normal to the earth's surface. For $t_0/T\ll 1$, the function (16) is linear in sec θ.

Of course, in practice T is not constant throughout the absorbing region of the atmosphere. Fortunately, this variation is not large nor is it a rapid function of height and a good approximation to $(t_\theta-t_0)$ is obtained by replacing T, in the above formulae, by

$$T_m=\int_0^h T(x)\alpha(x)dx\bigg/\int_0^h \alpha(x)dx. \quad (18)$$

T_m is called the mean temperature of the atmosphere averaged over the absorption along a vertical path and can be evaluated if the form of $\alpha(x)$ is known.

The cosmic noise is evaluated by comparing t_0 as calculated from (12) with that observed at the zenith. In the experiments such comparisons showed no systematic differences so that the cosmic noise is presumed to be negligible to these frequencies. However, the absolute accuracy of this result was not high (±20°K) for a number of experimental reasons. In any case, a small amount of cosmic noise if distributed uniformly in direction does not introduce much error in (11) for the values of t_0/T relevant here.

III. EXPERIMENTAL METHOD AND DATA

In the experiments three radiometers were operated simultaneously. Each was sensitive at a different wave-length in the region of the water vapor resonance; namely, at 1.00 cm, 1.25 cm, and 1.50 cm. All three had sensitive band widths, (total acceptance band widths to ½-power points), $\Delta\nu$, of 1.6×10^7 cycles/sec. During each run the antenna temperature at the zenith and the antenna temperature change with tipping was observed at the three wave-lengths.

The water vapor contents and temperatures of the atmosphere at various altitudes were taken from balloon and airplane soundings which were made at the time of day and in the geographical regions in which the radiometers were operated. The balloon soundings were kindly furnished to us by the AAF 26th Weather Region at Orlando, Florida. They employed commercially available "Radiosonde" equipment. In addition, the AAF personnel carried out several special flights over our Leesburg, Florida, location using airplanes equipped with the U. S. Army ML-313/AM psychrometer (wet and dry bulb thermometers in a housing which projects into the air-stream). Since these data were taken and analyzed by the AAF personnel we shall not discuss them here.

The microwave radiometer has been described elsewhere.[7] Suffice it to say that it consists of a superheterodyne receiver having a balanced, crystal mixer, a reflex-klystron local-oscillator, an intermediate-frequency amplifier of band width 8×10^6 cycles/sec. centered at 30×10^6 cycles/sec., a vacuum tube detector, and a narrow-band audio amplifier provided with a "lock-in" mixer. The antenna noise is intercepted by a tapered, rectangular horn connecting to a wave guide which carries the received signals to the receiver input. A slot in this wave guide admits a rotating, eccentric absorbing disk which periodically varies the attenuation in the wave guide from essentially zero to a large value. If then, the antenna temperature is different from the disk temperature (room temperature) the noise input to the receiver is modulated at the rotational frequency of the disk (30 cycles/sec.). Thus, the detected noise output from the intermediate-frequency amplifier contains a 30-cycle/sec. component which is amplified and beat against a 30-cycle/sec. signal (obtained from a generator attached to the shaft which drives the disk) in the lock-in mixer, producing a direct-current which is proportional to the difference between the antenna temperature and the disk temperature.

This use of a narrow channel at 30 cycles/sec. for the amplification of the noise signals has two great advantages over a simple scheme in which no modulation is employed and in which the direct-current component of the detected intermediate-frequency noise is observed. In the first place, vacuum tube amplifiers have inherent noise fluctuations which become very large as the frequency approaches zero. The modulation scheme enables one to avoid these large fluctuations and so to measure smaller changes in the antenna temperature. In the second place, the use of a 30-cycle/sec. channel enables one to stabilize the gain of the amplifier system accurately without degeneration of the 30-cycle/sec. signal. This is done by using the direct-current component of the detected noise from the intermediate-frequency amplifier to actuate a feedback system which removes all frequencies of less than 30 cycles/sec. and so stabilizes the amplifier against low frequency gain variations.

The intermediate-frequency band width is made as large as is consistent with small noise contributions by that part of the system, and the time constant of the output meter is made long (4 sec.), since under these conditions the output noise fluctuations are smallest and the useful sensitivity of the system for measuring changes in the antenna temperature is the greatest. The radiometers used had a useful sensitivity (antenna temperature change required to equal the r.m.s. noise fluctuations in the output meter) of 0.4°C. The direct-current output from the lock-in mixer is measured with a recording milliammeter which produces an inked trace which can be analyzed for the best fit with respect to the noise fluctuations over relatively long time intervals (say, 1 minute).

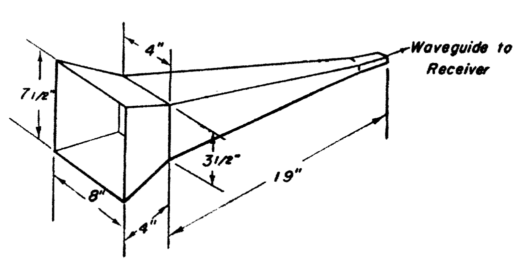

FIG. 2. Tapered, rectangular, horn antenna.

The antenna is shown in Fig. 2. The 4-inch by $3\frac{1}{2}$-inch aperture determines the antenna's directional properties. The flared section attached to this is to reduce the interception of radiation from the back hemisphere. This is necessary because such radiation originates from warm terrestrial objects.

The radiometer is calibrated by substituting for the antenna a matched wave guide termination which can be heated to known temperatures by means of a heating coil. This is called a hot load. Such calibrations were made before and after each series of observations on the atmospheric absorption. The sensitivity did not change more than 5 percent between such measurements.

As we have mentioned, the two essential observations are the measurement of the antenna temperature at the zenith and the measurement of the change in antenna temperature with tipping. A systematic procedure was used. First, an observation at the zenith, giving a deflection of say 250°C below the disk temperature. This deflection was then balanced out with a 30-cycle/sec. signal derived from the generator, and the gain of the audio amplifier was increased by a known amount. The apparatus was then tipped successively to 48.2°, 60.0°, and 66.5° remaining at a given position for say 1 minute, and this procedure was repeated in reverse order. The balancing signal was then removed, the zenith temperature measured, and the whole procedure repeated several times.[12] A typical tipping trace is shown in Fig. 3. The tipping deflections were converted to °C with the calibration constant

[12] It should be mentioned that it was found that some cumulus clouds were quite absorbing at the radiometer frequencies. For example, on April 11, 1945, a cloud was observed having absorption of 0.56 db, 0.36 db, and 0.25 db at the three wave-lengths 1.00 cm, 1.25 cm, and 1.50 cm, respectively (assuming a cloud temperature of 10°C, calculated from the ground-level relative humidity and temperature). Consequently, the radiometer antennas were never pointed at clouds during the atmospheric absorption measurements. Also, the sun was avoided for obvious reasons.

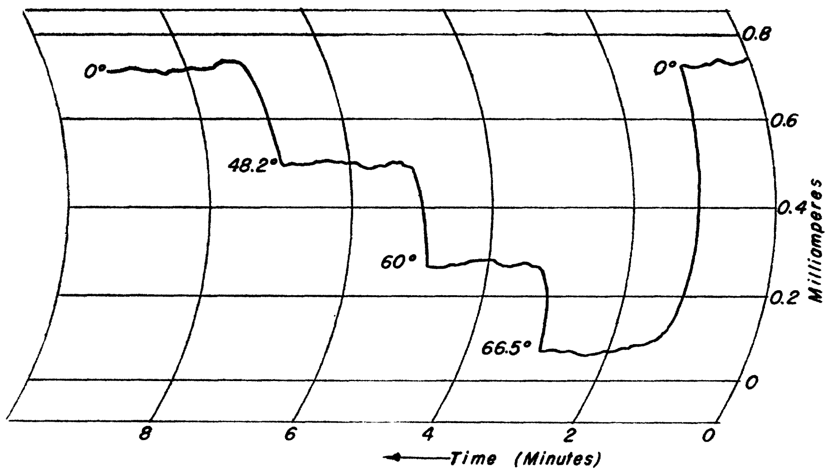

FIG. 3. Tipping trace with the 1.25-cm radiometer. (April 18, 1945.) Trace is labeled with tipping angles (measured from zenith).

deduced from the hot load calibrations[13] and the resulting deflections in °C, namely $(t_\theta - t_0)$, were plotted *vs.* sec θ as in Fig. 4. The best-fit straight line was then drawn through the three points $(t_{48.2}-t_0)$, $(t_{60}-t_0)$, and $(t_{66.5}-t_0)$ and the ordinate of this line at 60° was taken as $(t_{60}-t_0)$ in Eq. (17) from which the fractional absorption at the zenith, t_0/T, was calculated.[14,15] The values of T_m

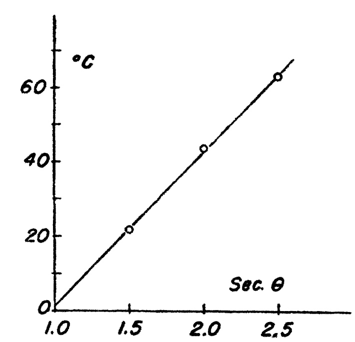

FIG. 4. Tipping deflection for the trace of Fig. 3. $t_\theta - t_0$ is expressed in °C.

used in Eq. (17) were deduced from Eq. (18) by a method which will be discussed in the next section.[16]

The collected absorption data are shown in Table I together with other data which will be discussed in the next section. The fractional absorption is expressed in decibels (db) in Table I. It is, therefore, the total absorption in db along a vertical path through the atmosphere, i.e.,

$$L_{\text{total}} = 10 \log_{10} (1 - t_0/T)^{-1} \text{ db.} \quad (19)$$

IV. REDUCTION OF DATA

In order to find the absorption coefficients for water vapor (db/km path/g H_2O/m^3/at standard temperature and pressure) from the measured fractional absorptions and the water vapor soundings a rather involved procedure is required. This is because of the following reasons: (a) the fractional absorption contains a contribution due to oxygen, (b) the water vapor is distributed with altitude and hence with pressure, (c) the thermodynamic temperature of the atmosphere is a function of the altitude.

[13] A small correction is applied to the calibration constant because of the difference in the absolute antenna temperature during the calibration and during the tipping. The effect of the feedback gain stabilizing system is to increase the radiometer gain by a small amount for the tipping as compared with that for the calibration, an amount which can be calculated accurately and which did not exceed 2 percent.

[14] From Eq. (11) it is seen that $(t_\theta - t_0)$ is not linear in sec θ for finite values of t_0/T. However, the best fit straight line still passes very near to $(t_{60}-t_0)$. It does, however, have a finite intercept at $\theta = 0$. Occasionally this intercept had to be adjusted to have the correct value by displacing the points of Fig. 4 up or down by equal amounts. This troublesome condition was caused by a change in radiometer zero setting in the tipping interval 0° to 48.2°. The effect was eliminated by increasing the mechanical rigidity of the wave guide circuits.

[15] Because of the finite directivity of the experimental antennas, the observed $(t_{60}-t_0)$ values differed from those given by Eq. (12), which was derived for a unidirectional antenna. This was a very small correction and was accurately calculated from the measured directivity of the antenna.

[16] A sufficiently accurate value of T is easily found by inspection of the water vapor sounding data since the range of variation of T is small compared with its mean value.

TABLE I. Total absorption, mean temperature, total water, equivalent water, and absorption coefficients computed from radiometer observations at Leesburg, Florida, in April, 1945.

	Total atmospheric absorption L_t (db) (including oxygen)			Mean temperature of atmosphere T_m (°K)			Total water G	Water vapor absorption L/G (db/kg/m²)			Equivalent water G_{eq} (kg/m²)			Absorption coefficient at S.T.P. $A(P_0)$ (db/km/g/m³)		
Date	λ=1.00 cm	λ=1.25 cm	λ=1.50 cm	λ=1.00 cm	λ=1.25 cm	λ=1.50 cm	kg/m²	λ=1.00 cm	λ=1.25 cm	λ=1.50 cm	λ=1.00 cm	λ=1.25 cm	λ=1.50 cm	λ=1.00 cm	λ=1.25 cm	λ=1.50 cm
Radiosonde runs																
April 8, 1945	0.29	0.77	0.41	285	284	284	26	0.0085	0.028	0.015	23	28	26	0.0095	0.026	0.014
9		.61	.30		284	285	20		.028	.013		21	20		.027	.013
10	.31	.65	.35	286	285	285	24	.010	.026	.013	22	26	25	.011	.024	.013
11	.29	.82	.41	287	287	287	21	.010	.037	.018	19	23	21	.011	.034	.018
12	.35	.85	.41	287	286	286	21	.013	.038	.018	19	24	22	.015	.034	.017
13	.29	.68	.37	289	287	288	26	.0085	.025	.013	23	28	26	.0097	.023	.013
14	.33	.77	.40	289	287	288	32	.0081	.023	.012	28	36	33	.0093	.020	.011
16	.33	.83	.42	292	291	292	27	.0096	.029	.014	24	29	27	.011	.027	.015
18	.44	1.07	.50	289	287	288	38	.0098	.027	.012	32	42	39	.011	.024	.012
20	.25	.52	.30	293	290	291	24	.0075	.020	.011	20	26	24	.0089	.019	.011
21	.29	.51	.28	290	286	288	25	.0088	.019	.010	22	28	25	.010	.017	.0098
23	.45	1.21	.62	289	287	288	36	.011	.032	.016	21	40	37	.012	.029	.016
24	.37	.89	.50	292	290	291	34	.0088	.025	.014	30	38	35	.010	.022	.013
25	.39	.85	.48	294	292	293	28	.011	.029	.016	24	31	28	.013	.026	.016
26	.41	1.10	.56	292	289	291	36	.0094	.029	.015	31	40	36	.011	.026	.015
27	.36	.82	.43	291	289	290	27	.011	.029	.014	24	30	28	.012	.026	.014
28	.23	.64	.33	293	291	292	26	.0062	.023	.012	23	29	26	.0070	.021	.011
													Averages	0.011	0.025	0.014
Airplane soundings																
16	0.33	0.85	0.43	288	287	287					24	29	27	0.011	0.028	0.015
27	.37	.85	.44	287	285	286					26	33	30	.012	.024	.014
28	.23	.66	.33	289	287	288					18	24	22	.0087	.026	.014
													Averages	0.011	0.026	0.014

The Oxygen Contribution

The first step is to subtract the oxygen contribution from the total fractional absorption. This is most easily done by simply plotting the observed fractional absorptions *vs.* the water vapor contents of the atmosphere (G in kg/m²) and to extrapolate this to zero water vapor; the absorption at that point being due to oxygen. This method is good when the experimental variation in G is large. For the Florida data this was not the case and another method was used. The measured L_{total} values at say 1.0 cm are plotted *vs.* those for say 1.25 cm, each point corresponding to a different day of observation. A straight line is drawn through these points. If then, it is assumed that the oxygen absorption varies in the theoretical manner,[5] namely, as $1/\lambda^2$, the intersection of this line with a line of slope $(1.00/1.25)^2$ gives the oxygen absorptions at the wave-length 1.00 cm and 1.25 cm. In principle, this procedure is more accurate than plotting L_{total} *vs.* G at each wave-length since the absolute amount of water vapor is not relevant and the only scatter is introduced by the different pressure variations of the water vapor absorptions at the three wave-lengths. The oxygen absorptions deduced in this manner are shown in Table II. The fractional absorption L for water vapor is obtained by subtracting the values of Table II from L_{total}.

The Absorption Coefficient

The water vapor contribution to the fractional absorption is made up of absorptions in various horizontally stratified layers each at a different pressure and temperature and each containing a different amount of water vapor. Unfortunately, the total absorption of these layers cannot be evaluated independently of the theory.

Before going into this matter, it is well to consider the following approximation. Consider the radio L/G in units db/kg/m². The total water vapor G is the mass of water vapor in a vertical column of one square meter cross section extending from ground level to infinity, and is calculated by simply summing the water vapor densities over the altitude. The ratio L/G, is, therefore, the observed absorption in db along a vertical path per kg/m² in the column. It is also the absorption in db along a 1 km path containing a unit density (1 g/m³) of water vapor. It is, therefore, neglecting the effects of temperature and pressure on the absorption, just the absorption coefficient (db/km/g/m³). Or, one may say that L/G is the absorption coefficient for some average atmospheric conditions. These values are shown in Table I.

The theory must now be invoked in order to reduce the data to an absorption coefficient at standard conditions (20°C, 1015 millibar). This involves assuming that the absorption coefficient depends on pressure in the theoretical manner. It is not necessary to assume the *absolute value* of the theoretical absorption coefficient. Let $A(\lambda, p)$ be the theoretical water vapor absorption coefficient[17] (db/km/g/m³) at a pressure p, a wave-length λ, and a temperature 293°K.[18]

$$A(\lambda, p) = (\text{const})\frac{v}{\lambda^2}\left\{\frac{2.65}{(1/\lambda-1/\lambda_0)^2+v^2} + \frac{2.65}{1/\lambda+1/\lambda_0+v^2}+9.65\right\}; \quad (20)$$

$v=pB/p_0$, where p_0 is standard pressure (1015 millibar) and B is one-half of the total line width (cm^{-1}) at half-maximum due to collision broadening at a pressure p_0 and temperature 293°K. The fractional absorption (in db) at wave-length λ(cm) along a vertical path is

$$L(\lambda) = \int_0^\infty \rho_w A(\lambda, p)dh \quad (21)$$

$$= \frac{10}{g}\cdot\int_0^{P_0} M(p)A(\lambda, p)dp, \quad (22)$$

where g is the acceleration of gravity (cm/sec.²), ρ_w is the water vapor density (g/meter³) at a height $h(km)$, and $M(p)$ is the corresponding mixing ratio (g H_2O/kg air). Let us define a relative absorption coefficient

$$\sigma(\lambda, p) = A(\lambda, p)/A(\lambda, p_0) \quad (23)$$

at each wave-length λ. Then

$$A(\lambda, p_0) = \frac{L(\lambda)}{\frac{10}{g}\int_0^{P_0} M(p)\sigma(\lambda, p)dp}, \quad (24)$$

$$= L(\lambda)/G_{eq}(\lambda), \quad (25)$$

where

$$G_{eq}(\lambda) = \frac{10}{g}\int_0^{P_0} M(p)\sigma(\lambda, p)dp \quad (26)$$

is called the *equivalent water*, being the mass (kg) of water vapor all at pressure p_0 in a column of 1 m² cross section which is required to give the observed absorption. $A(\lambda, p_0)$ is the desired absorption coefficient at standard conditions.

The integration (26) is carried out numerically using the observed distribution of water-vapor in the atmosphere and the function $\sigma(\lambda, p)$ which is calculated from (20) and (23) for assumed values of λ_0 and B. The values assumed were $B=0.12$ cm^{-1} and $\lambda_0=1.33$ cm. We shall see how these may be checked in a following paragraph.

The equivalent water and the absorption coefficient at standard conditions are given in Table I for the Leesburg, Florida, observations. The absorption coefficient can also be expressed as db/kg/m² being the absorption in db along a vertical path which contains 1 kg/m² of water vapor and throughout which the pressure is 1015 millibars.

The Mean Temperature

The mean temperature defined in (18) is seen to reduce to

$$T_m = \frac{\int_{P_0}^0 T(p)\sigma(\lambda, p)M(p)dp}{\int_{P_0}^0 \sigma(\lambda, p)M(p)dp}, \quad (27)$$

which can be integrated numerically by use of the meteorological data. These T_m values were used

TABLE II. Deduced O_2 absorption values in db for traversal of the entire atmosphere along the radio vector (Florida data).

λ	L (oxygen)
1.00 cm	0.07 db
1.25	0.04
1.50	0.03

[17] This formula is given in reference 6. The first and second terms give the resonance absorption (line at 1.3 cm) and the last term is the effect of the tails of the infra-red lines.

[18] The variation of $A(\lambda, p)$ with atmospheric temperature is neglected for two reaons. The analytic form of this variation is very complex and the range of temperatures observed at altitudes containing an appreciable amount of water vapor was small and fairly well distributed around 293°K.